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IN-MEDIUM ION MASS RENORMALIZATION AND LATTICE VIBRATIONS IN THE NEUTRON STAR CRUST

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The inner crust of a neutron star consists of nuclei immersed in a superfluid neutron liquid. As these nuclei move through the fermionic medium they bring it into motion as well. As a result their mass is strongly renormalized and the spectrum of the ion lattice vibrations is significantly affected. Consequently we predict that the specific heat and the lattice thermal energy of the Coulomb crystal at these densities are noticeably modified.

In the standard picture the inner crust of a neutron star consists of neutrons, protons and electrons. At low temperatures ($T \lesssim 1$ MeV) one expects the presence of atomic nuclei (ions) immersed in a neutron gas. At the densities: 10^{11} g/cm$^3 \lesssim \rho \lesssim 10^{14}$ g/cm^3 they form a crystal lattice stabilized by the Coulomb interaction. The electrons which are ultrarelativistic at these densities form a strongly degenerate, uniform gas. The detailed structure of this part of the star has been the subject of a considerable theoretical effort (.[1,2,3,4,5,6,7] and references therein). At the bottom of the crust the appearance of non-spherical nuclei, forming exotic structures, has been predicted. In this region besides the Coulomb interaction also the quantum effects, associated with a neutron scattering on nuclear inhomogeneities, play an important role, leading eventually to a disordered phase .[8,6].

In order to understand various processes associated with thermal evolution of a neutron star, the properties of Coulomb crystals in the inner crust have been studied by many authors (see e.g. .[9] and references therein). The plasma frequency of the system reads: $\omega_p = \sqrt{4\pi\rho_{ion}Z^2e^2/M}$, where ρ_{ion} denotes the ion density, Z is the proton number of an ion, and M is the ion mass. Since $\rho_{ion} = 1/\frac{4}{3}\pi R_c^3$, where R_c is the Wigner-Seitz cell radius then:

$$\omega_p = \sqrt{\frac{3Z^2e^2}{R_c^3 M}}. \qquad (1)$$

The quantities: Z and R_c, in the above equation, can be obtained from calculations based on a density functional method. However, the determination of the ion mass M requires a certain caution, since the nucleus is immersed in the fermionic

environment and its bare mass has to be renormalized to take into account the interaction with surrounding neutrons. If the neutrons outside a nucleus have formed a normal Fermi gas the problem would have been rather complicated.[10]. In the inner crust, however, it simplifies due to the fact that at low temperatures, $T \lesssim 0.1$ MeV, the neutron gas is predicted to be strongly paired. In this case one may apply the hydrodynamic approximation which describe the system by an irrotational velocity field. Although at finite temperatures there always exists a gas of quasiparticle excitations which form a normal flow, at temperatures much lower than the critical temperature its influence can be neglected.

We assume in the following that the nucleus has a uniform nucleon density ρ_{in} and fills a sphere of radius R. The neutron gas is characterized by the density ρ_{out}. We assume moreover that the flow velocity is below the critical velocity for loss of superfluidity.

In order to calculate the mass M of an ion, the Poisson equation for the velocity field has to be solved, both inside and outside a nucleus. The boundary conditions read:

$$\Phi_{in}|_{r=R} = \Phi_{out}|_{r=R}, \tag{2}$$

$$\rho_{in}(\frac{\partial}{\partial r}\Phi_{in} - \vec{n}\cdot\vec{u})|_{r=R} = \rho_{out}(\frac{\partial}{\partial r}\Phi_{out} - \vec{n}\cdot\vec{u})|_{r=R}, \tag{3}$$

$$\Phi_{out}|_{r\to\infty} = 0, \tag{4}$$

where Φ_{in} and Φ_{out} stand for the velocity field inside and outside a nucleus, respectively. The vector $\vec{n}$ is an outward normal to the surface of a spherical nucleus and r is a radial coordinate (the center of a nucleus is placed at $r = 0$). The velocity of the surface element was denoted by $\vec{u}$. The first equation stems from the requirement that the phase of the wave function of the superfluid system is continuous, the second one is a conservation of mass in radial flow, and finally the third one ensures that the correct asymptotic behavior for the velocity field is obtained. The knowledge of the velocity field allows us to determine the kinetic energy and corresponding effective ion mass. Consequently, the following expression for the mass of an ion being a subject of a translational motion can be derived:

$$M^{ren} = \frac{4}{3}\pi R^3 m\rho_{in}\frac{(1-\gamma)^2}{2\gamma+1}, \tag{5}$$

where $\gamma = \rho_{out}/\rho_{in}$, and m is a nucleon mass.

The above expression relates the mass of an ion to the density of the neutron gas. Note, that for a uniform system, i.e. $\gamma = 1$ the ion mass is equal to zero. Since the nucleon density vary substantially across the inner crust, one expects that the properties of the Coulomb crystal will be noticeably modified. Indeed, the plasma frequency in the inner crust changes as compared to the value calculated for the case: $\rho_{out} = 0$ (ion in the vacuum). Namely, $\frac{\omega_p}{\omega_p^{ren}} = \frac{|1-\gamma|}{\sqrt{2\gamma+1}}$, where ω_p^{ren} corresponds to the plasma frequency for the system with renormalized ion masses.

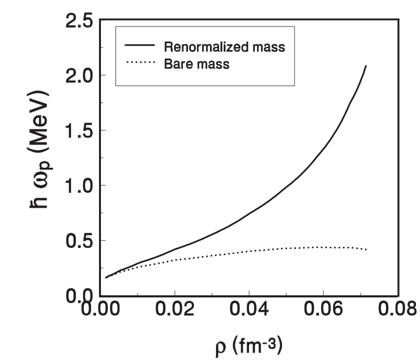

Fig. 1. Plasma frequency in the inner crust as a function of the nucleon density. Nuclear parameters were taken from .[4,11].

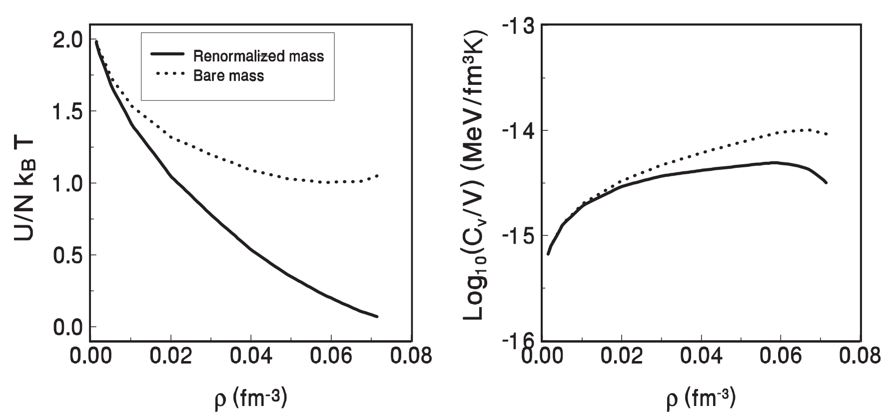

Fig. 2. The reduced lattice thermal energy: $u = U/Nk_BT$ (left figure) and specific heat per unit volume (right figure) at the temperature $T = 0.1$ MeV, as a function of the nucleon density for a bcc Coulomb crystal. N is the number of ions and k_B denotes the Boltzmann constant.

In the Fig. 1 we have plotted $\hbar\omega_p$ and $\hbar\omega_p^{ren}$ as a function of the nucleon density. One can notice large discrepancies between the value obtained using the bare nuclear mass and the renormalized one. It clearly indicates that the Coulomb crystal becomes more stiff at the bottom of the crust [a]. Consequently the density of the phonon spectrum is also modified as compared to the case of bare ion masses. The thermodynamic functions: the lattice thermal energy and the specific heat have been shown in the Fig. 2. One can notice that the renormalized values at a given

[a]One expects the phase transition to non-spherical nuclear phases to occur at the bottom of the crust. We disregard this effect in this paper.

density and temperature are smaller. In the limit of $T \to 0$ the analytic expressions for the ratios of lattice thermal energy U and specific heat C_v can be derived. Applying the phenomenological relations from the ref.[9] one gets:

$$\frac{U^{ren}(T=0)}{U(T=0)} = \frac{C_v^{ren}(T=0)}{C_v(T=0)} = \frac{|1-\gamma|^3}{(2\gamma+1)^{3/2}}, \tag{6}$$

where U^{ren} and C_v^{ren} denote the renormalized values of lattice energy and specific heat, respectively.

The following conclusions can be drawn from the presented results:

(1) There is a substantial renormalization effect of a nuclear/ion mass in the inner crust of a neutron star, due to the presence of a superfluid neutron liquid.
(2) Consequently, the phonon spectrum and thermodynamic functions of the Coulomb crystal are significantly altered.
(3) Thermal and electric conductivities of the inner crust, governed by the electron-phonon scattering are expected to be modified. In particular, the contributions coming from Umklapp processes have to be recalculated using the renormalized ion masses.
(4) There is clearly a need for consideration of other nuclear degrees of freedom like e.g. shape vibrations, which may significantly modify the electron transport properties.

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